Analysis & Probability/Algebra: Forest Mensurationist

Boise Cascade Corporation

Job Description: Use mathematics and statistics to inventory forest resources and predict the state of those resources in the future.

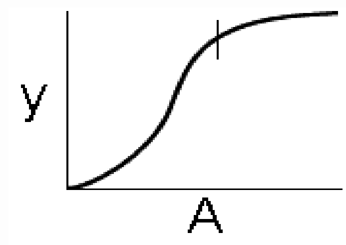
Problem:

In forestry, we can describe the relationship between the age of stand and the yield (timber volume) in that stand with the following "yield equation":

$y = ke^{-b/A}$, y = yield, e is the base of natural logarithms,

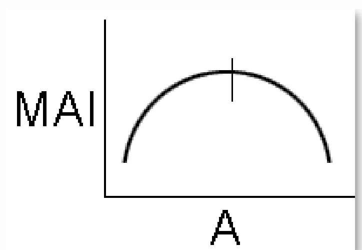
A = stand age, and k and b are constants that depend on the type of stand (species, geography, etc.)

The yield equation can be used to draw a yield curve, which looks like this:



If the yield at any age is divided by the age, we can derive the Mean Annual Increment (MAI), a measure of the rate of growth for that stand up to that age.

The age at which MAI is maximized is known as the "rotation age", the age at which the stand should be harvested, and then begin a new stand (a new rotation).



Here is the problem, which can be solved with calculus: Find the age at which MAI is maximized or use the definition of MAI to develop an expression for the rotation age.

[Hint: the yield equation shown above is popular because the rotation age = b; so the rotation age can be read directly from the yield equation]

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Solution:

Use calculus to show the age at which MAI is maximized is equal to b.

$$y = ke^{-b/A}, \text{ so } MAI = \frac{y}{A} = kA^{-1}e^{-b/A}$$

$$\frac{d}{dA}MAI = 0 \rightarrow \text{ peak of MAI curve}$$

$$\frac{d}{dA}kA^{-1}e^{-b/A} = 0 = kA^{-1}e^{-b/A}bA^{-2} + e^{-b/A}(-kA^{-2})$$

$$= \frac{ke^{-b/A}}{A}\left[\frac{b}{A^2} - \frac{1}{A}\right] \rightarrow \left[\frac{b}{A^2} - \frac{1}{A}\right] = 0$$

$$= \frac{1}{A}\left[\frac{b}{A} - 1\right] = 0$$

$$\rightarrow \frac{b}{A} = 1 \rightarrow b = A$$